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Fuzzy and Simulated Annealing on Site Location Problems

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Abstract

This paper addresses multiple objective site location problems in which there are imprecision in the constraints. The focus is on how imprecision can be dealt with fuzzy sets and then the simulated annealing (SA) algorithm is proposed, as a flexible and formalism independent tool, for solving linear and/or non-linear fuzzy site location problems. An illustrative example of a fuzzy multi-facility location problem for deciding the best location for two new police stations within a city is presented. The example is formalized both as a linear and as a non-linear problem in order to discuss the suitability of the proposed approach.

Keywords: fuzzy optimization simulated annealing, fuzzy multiple objective decision making.

1. Introduction

This paper proposes the use of the simulated annealing (SA) algorithm as a suitable method for solving fuzzy site location problems. Generally, site location problems are modeled unambiguously, a characteristic achieved through reduction and assumptions from reality which is typically fuzzy. This process of removing the conceptual vagueness in this type of problem also removes the intrinsic fuzziness of the human reasoning process. As the complexity of decision problems increases, the ability to define and model them in a precise way decreases, and the methods and tools to solve the problems are rarer. Hence, there is a need for more sophisticated tools that will reduce system complexity without losing the ability to identify the best possible solution.

Site location decisions, a type of optimization problem, are often characterized by uncertainty, subjectivity, imprecision and ambiguity in their parameters and in conflicting objectives. Therefore, their formulation and resolution are often complex. There are many types of locational problems as for example classical discrete plant site location, minimization of the costs of establishing a facility at some site, location of production facilities as to maximize profit, and stochastic facility locations in which demands, transportation costs and selling prices may be random [Louveaux, 1986] .

In general, three main categories of facility location decisions can be identified [Love, Morris et al., 1988] : a) continuous models where the facility location can be located in any place of the plane or of a subset; b) discrete models consisting of the application of mathematical programming to solve location-allocation problems; c) models applying graph and network theory. This paper only addresses the first and second models using a linear and non-linear multiple objective mathematical programming formalism.

The main concern of this paper is to present a new method for solving multiple objective site location problems which include some form of uncertainty within the model. Therefore, the example discussed is a fuzzy multiple objective decision problem of selecting the best location for two new police stations, which will serve certain areas of a city. The proposed solving tool is the simulated annealing algorithm, which is a stochastic algorithm with the physical analogy of melting a solid using high temperatures and then slowly lowering the temperature, until the system crystallizes/freezes and no further changes occur [Kirkpatrick, Gelatt et al., 1983] . There is a direct analogy between the melting process and the stretching of constraints, which is a direct result of their fuzzification, and between the crystallization process and the search for the "best" solution. Simulated annealing is considered a good tool for solving crisp optimization problems that are NP-complete [Kirkpatrick, Gelatt et al., 1983] and this paper extends the discussion to its application to fuzzy site location problems.

The paper is organized in six sections. First this introduction, second an introduction for fuzzy multiple objectives problems, third a brief description of the simulated annealing algorithm as well as its modification for handling fuzzy optimization problems and fourth an introduction to site location problems, their context and fuzzy environment. The fifth section presents an example of a fuzzy site location for two new police stations in Lisbon city and discusses the results obtained and the last section concludes this work.

2. Fuzzy Multiple Objective Decision Making

In general, multiple objective problems are concerned with the maximization or minimization of multiple, often conflicting, objectives subject to constraints representing limited resources. The aim is to maximize or minimize the objective functions while satisfying the problem constraints. Multiple objective problems are optimization problems coupled with heuristics to decide the importance or order of each objective. Essentially, optimization problems are based on unique solutions and the existence of complete information. The decision is usually to obtain the most promising solution, as, for example, combining the maximization of profits and the minimization of transportation costs.

Fuzzy optimization's main aim is to find the "best" solution (decision alternative) under incomplete information, i.e. imprecise information and/or vague resources limits. There are many forms of imprecision when dealing with fuzzy optimization. For instance, coefficient variables may not be known precisely (e.g. production time is *about* 2 hours to make a shirt) or there may be uncertain constraint limits (e.g. total production time should be less than *around* 200 hours). The current challenge is to be able to formalize models including both everyday language imprecision and uncertain data, and to solve them using quantitative methods capable of finding the best solution possible.

Fuzzy sets theory [Zadeh, 1965] provides the means to express imprecise concepts such as "about 2" or "more or less bigger or equal to 10", respectively an example of a parameter and of a resource limit. In fuzzy set theory they are usually called fuzzy numbers and its most common representation is done by triangular functions. Formally, a <u>fuzzy number</u> A in a real line R is a fuzzy set characterized by a membership function $\mu_A : \mathbf{R} \rightarrow [0,1]$ expressed as,

$$\mathbf{A} = \int_{\mathbf{x} \in \mathbf{R}} \mu_{\mathbf{A}}(\mathbf{x}) / \mathbf{x}.$$ where the integral represents a continuous function.

Many membership functions can be used to represent fuzzy sets. The most common are bell-shape, sinusoidal, triangular and trapezoidal. For example, a triangular fuzzy number has the following membership function:

$$\mu_{\mathbf{A}}(\mathbf{x})\begin{cases} 0 \quad \textbf{for} \quad \beta \leq \mathbf{x}; \mathbf{x} \leq \alpha \\ \dfrac{\mathbf{x}-\alpha}{\alpha} \quad \textbf{for}\ \alpha \leq \mathbf{x} \leq \rho \\ \dfrac{\beta-\mathbf{x}}{\alpha} \quad \textbf{for}\ \rho \leq \mathbf{x} \leq \beta \\ 1 \quad \textbf{for} \quad \mathbf{x} = \rho \end{cases}$$

The concepts of fuzzy goals and fuzzy constraints were first introduced by Bellman and Zadeh [Bellman and Zadeh, 1970] . The authors state that a fuzzy decision can be viewed as the intersection of fuzzy goal(s) and constraints - since they can be defined as fuzzy sets in the space of alternatives - and that the "optimal" decision (final solution) is the point at which the fuzzy goal(s) take its maximum membership value. This method is usually called the max-min approach. A systematic description and classification of these fuzzy problem types, methods and approaches proposed in the literature is given by Lai and Hwang [Lai and Hwang, 1994] .

Symbolically, fuzzy optimization problems with fuzzy coefficients and fuzzy parameters can be mathematically expressed by:

$$\begin{aligned} \max/\min \quad & Z = \tilde{C}x \\ & \tilde{A}x\{\geq,\leq,=\}\tilde{B} \\ & x \geq 0 \end{aligned}$$

where $\tilde{C}$ is the fuzzy costs vector, $\tilde{A}$ is the technological matrix containing the fuzzy parameters of the constraint variables and $\tilde{B}$ is the corresponding vector of fuzzy resource limits.

In general the fuzzification of single or multiple objective linear programming model includes four principal forms of imprecision [Fredizzi, Kacprzyk et al., 1991] (more subtle distinctions are made in [Lai and Hwang, 1994]):

1. Problems with fuzzy constraints. E.g. “the total travel time between warehouses should be *considerably less* than 4 hours ”.
2. Problems with fuzzy objectives (goals imposed on the objective functions). E.g. “the total cost for the project should be kept *well below* 100.000 Euros”.
3. Problems with fuzzy parameters (coefficients) on the variables of the objective functions or on the constraints. E.g. “the transportation cost per item (**x**) *is about* 10 Euros”.
4. Combinations of the above.

This paper discusses the first two cases of imprecision using a site location example. Following the symmetry concept which states that a fuzzy decision is viewed as the maximization of the intersections of any given goals and constraints, it therefore does not matter how many goals exist (for a complete literature review about fuzzy multiple optimization decision making see [Lai and Hwang, 1994]).

The resolution technique used with the simulated annealing algorithm, to solve our facility location example, can be summarized in five steps:

Step 1. In sequential order get an objective from the ordered (by preference) list of objectives.

Step 2. Obtain the maximum or minimum (optimal) value for the problem with that objective and all constraints.

Step 3. Incorporate into the set of constraints the equation obtained with the objective function equal to the max or min value obtained. The signal being a fuzzy equal to represent the acceptance of some deviation from the goal obtained.

Step 4. Go to step 1 until all objectives minus one are solved.

Step 5. Obtain the final optimal result for the last objective with all constraints.

Mathematically, the proposed steps are expressed by:

$$\begin{aligned} &\mathbf{Max\ /\ Min} \quad \mathbf{C_k} \\ &\mathbf{s.t.} \quad \sum_{\mathbf{j}} \mathbf{a_{ij} x_j} \left\{ \tilde{\leq}, \tilde{\geq}, \tilde{=} \right\} \mathbf{b_i} \\ &\qquad \mathbf{Z_k} \tilde{=} \mathbf{C_k} \\ &\qquad \mathbf{x_j} \geq 0. \end{aligned}$$

where k = 1, ..., n-1; i = 1, ..., m ; j = 1, ..., l; C_k is the max or min of the objective function Z_k and the signals tilde represent the fuzzy equations limits.

Most of the literature about fuzzy optimization is concerned with the fuzziness at the modeling level, from goal preferences to goal priorities. The majority of authors follow the maxmin approach, using an equivalent crisp model of maximizing the aggregation of the minimum deviations from the model constraint levels [Zimmermann, 1978] . In summary, they do not propose a fuzzy solution but an equivalent crisp one (overview in [Lai and Hwang, 1994]). However, some attempts have been made to obtain the benefits of using algorithms appropriate to fuzzy optimization such as simulated annealing ([Connoly, 1992] ; Connoly, 1990] ; Eglese, 1990] ; Ishibuchi, Tanaka et al., 1994]).

It should be noted that one of the advantages of using the SA algorithm as a resolution procedure is generality and independence of the problem formalization. Therefore, the SA seems a suitable tool to solve any type of fuzzy linear or non-linear optimization problems. A good application example of a crisp non-linear problem, using the SA algorithm, is the quadratic assignment problem described in [Connoly, 1990] . Other examples of fuzzy multiple objective problems using different resolution methods are described in [Bit, Biswal et al., 1993] ; Fredizzi, Kacprzyk et al., 1991] ; Lai and Hwang, 1994] . Both the linear and the non-linear site location example discussed in this paper illustrate the generality of our approach.

3. Simulated Annealing

This section summarizes the adaptation of the simulated annealing algorithm to solve fuzzy optimization problems [Pires, Pires et al., 1996] . In general, the simulated annealing is a stochastic algorithm used for optimization problems where the objective function corresponds to the energy of the states of a solid [Eglese, 1990] . The algorithm physical analogy refers to the process of melting a solid and then by lowering the temperature slowly finding a crystallizing point. The SA algorithm requires the definition of the neighborhood structure as well as the parameters for the cooling schedule. Further, the SA is a good tool for solving optimization problems considered NP-complete, i.e. computationally inefficient since the search for the optimum is an exponential function of the size of the problem [Kirkpatrick, Gelatt et al., 1983] .

The implementation of the SA algorithm used here handles the fuzzification of constraints and was proposed by Ribeiro and Moura-Pires [Ribeiro and Moura-Pires, 1997] . The two authors also implemented the resolution with the SA algorithm for the maxmin method, equivalent to Zimmermann´s proposal [Zimmermann, 1978], by maximizing the aggregation of the membership values of the goals and constraints. In this paper we only address the simulated annealing algorithm steps for the fuzzification of the constraints:

```
Set Nr = number of constraints;
Select a initial state x ∈ X;
Select a initial temperature T > 0;
Set temperature change counter t = 0;
Repeat
    Set repetition counter n =0;
    Repeat
        Generate state y a neighbor of x;
        k := 1
```

```
        Repeat
            μ(k) = membership value of the constraint Rk(y);
            k := k+1;
        until k > Nr or μ(k-1) = 0;
        If  μ(k-1) ≠ 0
            then
                Aμ = aggregation(μ(1), …, μ(Nr));
                Calculate δ = Z(y) - Z(x);
% Minimization
                If δ < 0 then   x:= y;
                    else If random(0,1) < exp(-δ/(Aμ*T)) then x := y;
        n := n+1
    until n = N(t)
    t := t + 1
    T = T(t)
until stooping criterion true.

Note: for a Maximization problem:
                If δ > 0 then   x:= y;
                    else If random(0,1) < exp(δ/(Aμ*T)) then x := y;
```

The variable k is a counter for the number of constraints. It is used for calculating the memberships of each constraint μ(k). Aμ represents the aggregation of the memberships of the fuzzy constraints. Any aggregation operator could have been considered, here the t-norm -min- is used. The variable δ shows the difference between the previous objective and the new candidate. This represents the old and new energy states difference. The probability function of moving to a smaller energy state (the goal) is given by the exponential of δ divided by the Aμ and the control parameter T. The smaller the temperature, T, the less probable any change will occur. In this version a finer selection process is performed by multiplying the membership Aμ by the control parameter T to select the best goal Z. The N(t) represents the number of neighbors generated as possible solutions to be tested. The T(t) is the decreasing function of the temperature. Usually, this decreasing factor is around 0.9.

It should be noted that other local search methods could also have been considered such as, for example, the Tabu Search algorithm [Glover, 1989] [Glover, 1989] . However, as far as we know, there are no comparative studies about search algorithms for fuzzy site facility decision problems and since such comparison is a research issue in itself it should be developed in the next future. An interesting comparison between the tabu search and simulated annealing algorithms for crisp clustering and partitioning problems is addressed in the literature [Amorim, Barthelemy et al., 1992] .

In summary, the SA algorithm can solve both linear and non-linear optimization problems. This characteristic shows this algorithm is a very flexible and a formalism independent tool for solving any type of fuzzy site location problems.

4. Introduction to Fuzzy Site Location Problems

Locational choices commit firms to long lasting costs, employment and marketing patterns. No optimal decision process can ensure an optimal location is chosen; avoiding a troublesome (or even disastrous) location may be more important than trying to find the ideal site. Among the major factors influencing location decisions are the incoming raw materials, human resources and capital. Major outgoing factors include the market and the environment.

Location decisions may involve so many aspects that even a systematic approach is not enough to deal with imprecision contained in some factors, such as costs, slight deviations and sizes because the trade-offs are usually accepted. For instance lets look at the imprecise nature of factors such as political or social ones. Lets say that for social reasons it might be better to select a site with good transportation, but this might prove to have much higher costs than if the site is outside a high price marketable zone. Regarding political factors, lets say the government offers subsidies to organizations which select a site in a low populated area, this might be a disastrous selection in terms of market, because it also implies less costumers and higher transportation prices to populated zones. How do we measure these factors? The usual process is to estimate the parameters for the variables involved, the resource limits and their respective relations. Many of these estimations are just this, estimations that the decision maker is more than willing to trade-off for attaining a more balanced solution. The important aspect is to find forms of expressing imprecision or desired flexibility on the fuzzy parameters or resources of the problem, and what this paper proposes is to use fuzzy sets for that representation and modeling.

In general a multi-facility location problem with linear constraints can be formalized as follows [Love, Morris et al., 1988] :

$$\underset{\mathbf{X}}{\textbf{Minimize}} \quad \mathbf{Z(X)} = \sum_{\mathbf{i=1}}^{\mathbf{m}} \sum_{\mathbf{j=1}}^{\mathbf{n}} \mathbf{w_{1ij} d_p (X_i, a_j)} + \sum_{\mathbf{i=1}}^{\mathbf{m-1}} \sum_{\mathbf{r=i+1}}^{\mathbf{m}} \mathbf{w_{2ir} d_p (X_i, X_r)}$$

$$\mathbf{s.t.} \quad \mathbf{AX \geq B}$$

where

m is the number of new facilities to be located,

n is the number of existing facilities,

w_{1ij} converts distance between new facility i and existing facility j into cost and $w_{1ij} \geq 0$,
w_{2ir} converts distance between new facility i and new facility r into cost and $w_{2ir} \geq 0$,
$X=(X_1,\ldots,X_m)$ is the vector of the new facility locations,
$X_i=(x_{i1},x_{i2})$ is the location of the new facility i,
$a_{ij}=(a_{j1},a_{j2})$ is the location of existing facility j,
$d_p(X_i,a_j)$ is the distance between new facility i and existing facility j,
$d_p(X_i,X_r)$ is the distance between facility i and new facility r,
A is the technological matrix of variable parameters, and
B is the vector of the resource limits.

According to the distance function used $d_p(X_i,a_j)$ the problem can be either linear or non-linear, such as the cases of using respectively Manhattan or Euclidean distances. Many models and resolution methods have been discussed in the literature for the crisp case, but it is out of scope of this paper to discuss this subject any further (see for example [Love, Morris et al., 1988])

The objective of this paper is to show that instead of being constrained by rigid formalizations and limiting parameters or resources a more flexible approach can facilitate the whole process both at the modeling level as well as at the solving level. An interesting fuzzy facility location problem has been discussed by Chung and Tcha [Chung and Tcha, 1992] . Their approach fuzzifies the amount to be supplied to each demand site, by considering a minimum goal for each variable, and also fuzzifies a goal for the objective function. The proposed fuzzification lacks generality and resolution independence since it cannot solve the existence of fuzzy parameters and is dependent on the maxmin method [Bellman and Zadeh, 1970] for resolution. Further, to assume goals for decision variables does not seem very realistic because these are what we are trying to solve. The author's perspective is not really to allow a flexibilization of a crisp problem but to define fuzzy goals.

5. A Fuzzy Site Location Example

The example presented in this section trades realism for simplicity since only considers fuzziness in the constraints limits. However, it contains all the main elements of complex problems, multiple objectives; multi-facilities; non-linearity; and fuzzy constraints. Two formalizations are tested to assess how flexible and adaptable the SA algorithm is for solving this type of problem. One uses a linear distance between locations, Manhattan distance, depicted in Figure 1, while the second uses the Euclidean distance, i.e. a non-linear formalism, depicted in Figure 2. The rationale for using these two formalizations is to show how adaptable the proposed algorithm is to different models for fuzzy site location problems.

Consider the objective of selecting the approximate locations for two new police stations, which will serve the inhabitants of 4 neighborhoods, located high-crime areas of Lisbon City. The background to the example could be the complaints, by local residents, of the time taken by police to answer distress calls and the lack of frequent street patrols mainly due to distance from the nearest police station. A committee was nominated to decide the best locations for the two new police stations. The requirements defined by the committee are the minimization of their distances from the 4 neighborhoods and the maximization of the distance between the two police stations to improve police efficiency and avoid overlapping of police patrols with the consequent increase in running costs. Due to rising construction costs in the center of the city, two constraints were added to prevent, as much as possible, the selection of costly locations.

To simplify the discussion only point coordinates are considered, the central points of the four neighborhoods and the point coordinates for the new police stations. This simplification agrees with the type of imprecision addressed in this paper, since the main concern in a fuzzy environment is to obtain an approximate result and, hence, seeking the "best" approximate locations for the two police stations under vague conditions. The final decision will have to consider other issues such as availability of buildings, costs of construction and many other parameters related with the exact location of the two police stations.

Formally, lets consider the following approximate center coordinates for the 4 neighborhoods: (1) a_1=2 b_1=8; (2) a_2=6 b_2=6; (3) a_3=1 b_3=1; (4) a_4=1 b_4=4. The distance from the point coordinates for each police station is given by $|x-a_i| + |y-b_i|$, where x and y represents the location coordinates to choose. The first formulation for the problem uses a modular linear approach (Manhattan distance):

$$
\begin{aligned}
&\mathbf{Min} \quad \mathbf{G}_1 = \sum_{\mathbf{i}} \left(|\mathbf{x}_1 - \mathbf{a}_\mathbf{i}| + |\mathbf{y}_1 - \mathbf{b}_\mathbf{i}| \right) \\
&\mathbf{Min} \quad \mathbf{G}_2 = \sum_{\mathbf{i}} \left(|\mathbf{x}_2 - \mathbf{a}_\mathbf{i}| + |\mathbf{y}_2 - \mathbf{b}_\mathbf{i}| \right) \\
&\mathbf{Max} \quad \mathbf{G}_3 = |\mathbf{x}_1 - \mathbf{x}_2| + |\mathbf{y}_1 - \mathbf{y}_2| \\
&\mathbf{s.t.} \\
&\qquad \mathbf{x}_1 + \mathbf{y}_1 \tilde{\geq} 8 \\
&\qquad \mathbf{x}_2 + \mathbf{y}_2 \tilde{\leq} 6 \\
&\mathbf{x}_1, \mathbf{x}_2, \mathbf{y}_1, \mathbf{y}_2, \mathbf{a}_\mathbf{i}, \mathbf{b}_\mathbf{i} \geq 0 \qquad \mathbf{i} = 1, \ldots, \mathbf{n}
\end{aligned}
$$

Figure 1. Manhattan distance formulation

The second formulation approach, the non-linear Euclidean distance is:

$$\mathbf{Min} \quad G_1 = \sum_i \left((x_1 - a_i)^2 + (y_1 - b_i)^2 \right)$$

$$\mathbf{Min} \quad G_2 = \sum_i \left((x_2 - a_i)^2 + (y_2 - b_i)^2 \right)$$

$$\mathbf{Max} \quad G_3 = (x_1 - x_2)^2 + (y_1 - y_2)^2$$

$$\mathbf{s.t.}$$

$$x_1 + y_1 \tilde{\geq} 8$$

$$x_2 + y_2 \tilde{\leq} 6$$

$$x_1, x_2, y_1, y_2, a_i, b_i \geq 0 \qquad i = 1, \ldots, n$$

Figure 2. Euclidean distance formulation

As can be observed the difference in the two formulations is in the type of objective equations. In Figure 1 objectives are linear while in the second formulation (Figure 2) they are non-linear. One asset of using the SA algorithm to solve this type of problems is its independence from the equation type. The SA algorithm is not constrained by standard formalizations, it can solve all types of crisp and fuzzy optimization problems [Connoly, 1992] ; Eglese, 1990] .

In order to represent the fuzzy constraints, the functions selected were triangular functions, because they are widely used, due to its simplicity and linearity. Graphically they are represented by:

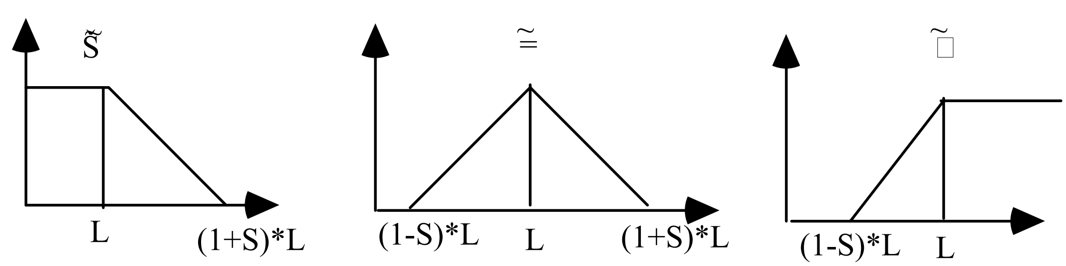

Figure 3. Fuzzy membership functions used in the location problem

The slope considered for the construction of the triangles was => S=10%; L represents the constraint limits, since a fuzzified constraint is semantically read as : "approximately bigger (or smaller) than L".

With the use of fuzzified constraints we are able to obtain two types of results, one is the best solution achieved, the second is the degree to which the constraints were satisfied. The degree of satisfaction is given by the membership value obtained with the triangular functions depicted in figure 3. The further a constraint is relaxed to achieve a better result, i.e. the further away is from its center, L, the smaller degree of satisfaction that is obtained for the constraint. In our problem we stipulated a minimum level of satisfaction, threshold, for the constraints of 60%. Many fuzzy approaches to linear programming define goals for the problems and then transform the problem into an equivalent one of maximizing the degree of satisfaction for both constraints and goals [Lai and Hwang, 1994] ; Zimmermann, 1978]. Here, we concentrated on obtaining the best result for the coordinates in addition to considering a minimum level for constraint satisfaction of 60%.

The solutions found for the first formalization (Manhattan distance), using the SA algorithm indicate the following approximate location for the two police stations:

X_1=2.0 Y_1= 3.7

X_2=1.4 Y_2=6.3

$\mu(c_1)$= 1 $\mu(c_2)$= 0.6 (membership values representing the satisfaction of constraints 1 and 2)

The solutions found for the second formalization (Euclidean distance) were:

X_1= 1.6 Y_1= 4.2

X_2= 3.1 Y_2= 5.2

$\mu(c_1)$= 1 $\mu(c_2)$= 1

Graphically, the solutions can be observed in Figure 4.

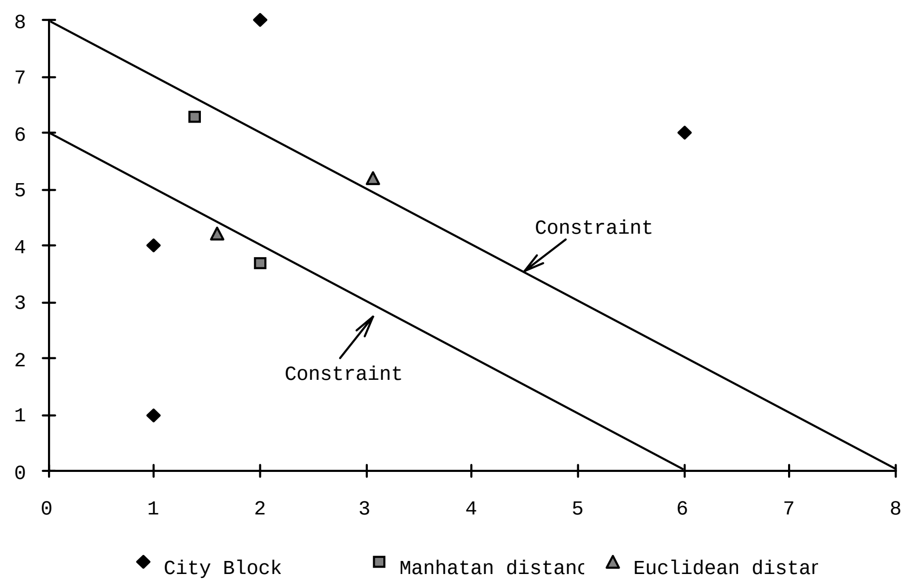

Figure 4. Results with both Manhattan and Euclidean distances

As can be observed the results obtained with the Euclidean distance have a better degree of satisfaction for the two constraints $\{\mu(c_1)=1,\ \mu(c_2)=1\}$, while the solution obtained with the Manhattan distance is only achieved by stretching constraint C2. The later implies that in a crisp environment either the problem was infeasible or the result would be much worse than the one obtained.

Even with this simple problem, two important conclusions can be drawn from the fuzzy solutions obtained, first the Euclidean distance formalization seems more appropriate, since it finds a solution without violating the constraints, and, second the constraints degree of satisfaction provide a discriminating feature between solutions obtained. Another important benefit of calculating the constraints satisfaction level is the knowledge about the trade-offs that should me made on the constraints limits to achieve the better results. This is a simulation ability cheered by most decision makers who, usually, are willing to make trade-offs in exchange for additional benefits. In summary, to allow deviations from the constraints limits (fuzzy constraints) has the twofold ability of always finding solutions and accepting the imprecise nature of the majority of site location problems.

The fuzzy results obtained were also compared with a crisp version of the problem, using the simplex method. The crisp version was modeled using the goal programming method and the linear Manhattan distance, where the positive and negative deviations represented the transformation of the modules. The result obtained was X1=2, Y1=4 and X2=6, Y2 =6, clearly worse than any fuzzy solution since one police station will only serve one neighborhood (6,6) because it is too far away from the other neighborhoods. An important aspect to emphasize is that if the problem was infeasible (in the crisp version) the fuzzy version is always feasible, depending on the trade-off the decision maker is willing to exchange for constraint satisfaction versus objective satisfaction.

Concluding, fuzzy site location problems can be solved by the SA algorithm independently of their specific formulation. This method provides a freedom to the decision maker to use any model s/he finds appropriate for the specific case. Of course, the decision maker must also take into consideration that, since the initial conditions are fuzzy, the results obtained are the best solution identified and not a crisp mathematical optimum. This does not appear to be a problem for the majority of site location problems, since the decision to build in a specific location or at a small distance away usually does not make much difference.

In addition, the approach proposed is sufficiently general to allow consideration of other forms of fuzziness (as described in section 2) as well as other forms of models [Love, Morris et al., 1988]. The results obtained with the SA algorithm are proving valuable enough to merit further investigation, and research is already in progress to extend this approach to other types of optimization problems.

6. Conclusions

The increasing complexity of site location problems requires methods that allow the inclusion of some form of fuzziness or imprecision, in the models and in the methods, to solve them. This paper discusses a flexible approach that can be used to solve fuzzy multiple site location problems. The approach combines the flexibility of a fuzzy context with the flexibility and independence of the simulated annealing algorithm, thus providing good solutions without loosing consistency.

The main advantage of using this approach, as the example shows, is freedom for the decision maker to choose whichever model is appropriate for the specific problem. A further advantage is the freedom given by working in a fuzzy environment, which allows the inclusion of imprecision or vagueness in the parameters and coefficients of the problem. An example of a site location problem for police stations was provided to discuss these issues.

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