Draft of paper In: Encyclopedia of Microcomputers. Edited by: Allan Kent, James G. Williams. Vol. 22 Supplement 1, Marcel Dekker, Inc, (1999) 335-346

UNCERTAINTY IN DECISION MAKING

Dr Philip Powell, Warwick Business School, UK and Dr Rita Almeida Ribeiro, Universidade Nova de Lisboa, Portugal

I. The Nature of Decision Making and Uncertainty

The objective of a decision is to make a choice and the objective of decision theory is to study how decisions are made or ought to be made. What makes decisions difficult, however, is the existence of doubt, conflict and uncertainty as to the outcomes or selection criteria.

The process of decision making is usually presented as series of steps. In the classical model [1] it was three, but sub-division gives the following six [2]:

define the problem
identify the criteria
weight the criteria
generate alternatives
rate each alternative in each criterion
compute the optimal decision

Put like this, decision making appears quite straight-forward. However, all decision situations require some sort of choice between alternative actions in order to try to achieve a desired outcome. Infrequently, there is certain knowledge of the outcomes, yet, more often, certainty is unlikely and the decision maker will be operating under uncertainty. This uncertainty may range from the almost certain to the completely uncertain and may arise either from incomplete knowledge about the world, as when the outcomes of a decision depend on future states, or from lack of knowledge about oneself, when it is not known which outcomes are more satisfying. For example, investing in fixed interest US Government securities gives an almost certain annual return, though there are slight default risks, inflation effects and so on. Conversely, investing in blue-sky research for a cure for the common cold has an extremely uncertain outcome.

In order to make rational decisions, the decision maker needs to be able to rank the alternative choices, that is, establish a preference ordering of the alternatives. A rational decision is based on two important principles [3]: transitivity - if alternative a is preferred to alternative b and b is preferred to c then alternative a is preferred to c; asymmetry - if the decision maker prefers alternative a to b than s/he cannot also prefer b to a. The transitivity and asymmetry principles are the basis for establishing any preference ordering which, depending on the context, can be partial or complete. Typically, a complete ordering means that all alternatives can be ranked, while a partial ordering occurs when the decision maker is indifferent between two or more alternatives. The latter is usually denoted as weak or indifference preference ordering [3].

There are two main categories of decision theories [4]. Descriptive decision theory which is concerned with how to describe and explains the decision maker's choices, and normative decision theory which attempts to prescribe which is the best (or optimal) decision to be made.

II. The Contexts of Decision Making

In general the decision making process can be classified into three main contexts (or environments), depending on the state of knowledge about the alternative courses of action [3, 5]:

A. Decision making under *certainty* (complete information).

This assumes that complete information about the outcome of each course of action is known and therefore knowledge is structured and deterministic. Decision making consists of optimizing a utility function and since there is no random or chance process involved, the choices are also called riskless. Under conditions of certainty only one state of nature is possible or the consequences of any decision are robust if variations do occur. It should be pointed out that decision making under certainty does not necessarily mean that the decision will be easy. It may be very complex to calculate the alternative outcomes, or very expensive to acquire the necessary data or difficult to compare the alternatives, or, indeed, very difficult to implement the decision once made.

B. Decision making under *risk* (less than complete information).

This is also known as a probabilistic or stochastic decision situation. A technical distinction is usually made between risk and uncertainty. Risk refers to a situation in which there is a possibility of suffering loss, while uncertainty is the state of not knowing something definitely. Thus risk occurs in situations when probabilities can be calculated on the basis of an objective classification of instances while uncertainty refers to situations where no objective classification is possible [6]. However, in many cases the two terms, risk and uncertainty, are used interchangeably since the process of decision making often uses non-statistical or subjective probability assessments.

Here, the decision maker evaluates the outcomes or the state of natures' likelihoods since each has a probability and/or expected value of occurrence, i.e. the degree of risk assumed can be assessed. An example might be to consider taking a loan from bank A or bank B which offer different conditions and for which trade-offs must be taken into account.

C. Decision making under *uncertainty* (least information).

The decision maker must consider situations in which several outcomes are possible for each course of action and existing information is vague (or fuzzy), incomplete or missing. An example might be to hire an employee based on an interview where you are not sure what qualities are desirable in the candidate nor how well an interview will reveal those qualities. For instance, hiring a new receptionist based on an interview which attempts to assess criteria such as good communication skills or the pleasantness of voice.

Total uncertainty, on the other hand, not only means that it is difficult to predict the consequences of decisions but that even the possible alternatives and, further, the data are extremely uncertain. Indeed, in very uncertain cases the decision maker may not even know what s/he wants.

In the real world, unfortunately, very few decisions can be made under total certainty, and, luckily, few have to be made under total uncertainty. The majority of the problems faced by decision makers are complex and ill-structured. Hence most decision makers operate in a world which is somewhat uncertain and thus it is necessary to review the different ways that this uncertainty may be handled.

III. Sources of Imprecision

As stated above, uncertainty may arise from incomplete knowledge about the world, or from lack of knowledge about oneself. The main sources of imprecision in decision making are usually attributed to the natural world [7]: unreliable sources of data and information; abundance of irrelevant data; imprecision of natural language; lack of understanding; faulty sensory mechanisms (vision, tactile, taste etc.); conflicting or complementary sources of facts; hidden variables producing apparent randomness (stochastic events); energy/costs of collecting information. These sources of imprecision can be grouped into three main categories [8]:

A. *Incompleteness*

Incompleteness can arise in situations lacking some alternatives, attributes or having insufficient data to stipulate a constraint limit. Further, sometimes the cost of obtaining a crisp measure is too high and an approximation will suffice; for example, the expected revenue for 1997 is 'around' $10 million. This is an uncertain sentence and the incompleteness is shown in the uncontrollable variables that affect revenue and in the difficulty of making a precise forecast.

B. *Fuzziness*

This may arise from difficulties in quantifying concepts for attributes, criteria and constraints. Usually, these concepts are expressed by linguistic terms such as good or fair applied to an attribute. Fuzzy data is qualitative and usually obtained by subjective judgements from individuals; for example, inflation is high. This example uses the fuzzy descriptor high to 'classify' the attribute inflation. It is not incomplete, it is only a qualitative proposition. Conversely, a crisp data example is that inflation is 4%.

C. *Illusion of validity*

The illusion of validity arises from detecting erroneous outputs such as large deviations from 'expected' solutions or selecting non-relevant alternatives perhaps because the match was performed by using inappropriate stereotypes. The term illusion of validity comes from Tversky and Kahneman [9], though the original sense only considered judgement and selection of outcomes under uncertainty, yet in abstract this and the concept employed here are similar. For example, the revenue was $5 million when the expected value was $10 million. The illusion of validity could have derived from using only a reduced series (e.g. prior year, 6 month sales) to produce the forecast or by not using the appropriate technique to derive it. In this case, imprecision is due to insufficient input which resulted in incorrect output.

Iv. Uncertainty Modelling and Representation

Essentially, the decision maker may handle uncertainty in a quantitative or a qualitative way. The choice as to which is best will depend on many things including the nature of the decision, the availability and type of data, the skills and, indeed, the preferences of the decision maker. In may be possible to disaggregate the decision into components and use different techniques for different parts, or try different approaches to the same component to get some triangulation on the solution. Of course, a further alternative is to ignore the uncertainty and tackle the problem as if all elements were certain.

Sources of uncertainty may be economic, political, social and environmental. They may be uncertain to the project in hand, uncertain to the business, or the global economy or whatever. Klir and Folger [10] distinguish two distinct forms of uncertainty - vagueness and ambiguity. Vagueness is associated with the difficulty of making sharp or precise distinctions in the world while ambiguity is associated with one-to-many relationships, that is, situations in which the choice between two or more alternatives is left unspecified.

The way in which any decision involving uncertainty can be tamed is by first identifying the variables present and how they are related, and then by assessing the impacts of the uncertainties involved. All the approaches discussed have these themes underlying them.

A. Quantitative Approaches

The traditional way to assess uncertainties about the future is by forecasting. This involves attempting to predict the future values of uncertain variables and then make decisions based upon the most likely future. This type of approach has proved less useful as the environment has become more turbulent and, as discussed below, not all forecasts are quantitative. More recently, risk analysis has been used to make explicit the recognition of uncertainties and the modelling of these by probability density functions.

1. Forecasting

Perhaps the most common way to address uncertainty in decision taking is to forecast the future. Forecasts attempt to predict. The possible ways to forecast are many and varied, ranging from sophisticated time series analyses and use of neural networks to simple guesses based on intuition. It is beyond the scope here to discuss forecasting in depth, however an appreciation can be gained by looking at the types of forecast available. Cooke and Slack [11] provide a framework which classifies the techniques by their objectivity or subjectivity on the one hand and the extent to which they are causal or not on the other. Subjectivity implies the use of intuition or judgment. Causal methods attempt to predict on the basis of factors which influence the variable to be predicted. Non-causal methods, on the other hand, use past values of a variable to predict its future values.

Objective causal methods include regression approaches and econometric models, while time series are examples of approaches which are objective but not causal. Subjective causal methods are those that use single or group expert opinion to identify the underlying factors which cause events to happen. Subjective non-causal methods also use opinion but do not reflect on the causes of events but just on the outcomes and their values. It is fair to say that while objective methods have become much more sophisticated as both the techniques have improved and computing power has allowed more data to be considered, the recent moves in business have been to the softer, qualitative approaches such as the development of scenarios.

2. Risk Analysis

As identified above, risk analysis models uncertainty by obtaining a forecast of the variables' probability density functions. Monte Carlo simulation can be used to approximate the probability distribution of the outcome by taking a controlled random sample of combinations of the variables in such a way that the probability distributions of the variable values are reflected in the result obtained [12]. As in other cases, the potential worth of a risk analysis is only as good as the model upon which it is based. If major elements are omitted or the interactions of the variables incorrectly specified (for example, dependent variables modelled as independent) then the model outputs will be seriously flawed.

3. Decision matrices

Decision matrices are ways of modelling relatively straight-forward decisions under uncertainty in such a way as to make explicit the options available, the states of nature pertaining to the decision and the decision rules used to choose between the options [11]. The usual format is to lay out the problem as a matrix with one axis showing the options and the other the states of nature that might obtain. At the intersection of each state and option, the outcome is enumerated. Assuming that it is possible to enumerate these outcomes, there is still the problem of deciding which option is preferred. A number of rules have been suggested for this, the most common being the optimistic decision rule, the pessimistic, the regret and the expected value decision rule.

As the name suggests, the optimistic decision rule maintains that the preferred alternative is the one which yields the best available outcome. It is sometimes referred to as the maximax option as it picks the maximum option for the maximum state of nature. In contrast to this optimistic approach, the pessimistic decision rule suggests that the preferred option is to pick the one which provides the best of the worst outcomes. This is a maximin approach as it maximizes the minimums on offer. The regret approach essentially poses the question of how upset the decision maker would be, in retrospect, if they chose a particular option compared to the best option under the state of nature which obtains. The goal is minimax regret, that is, to minimize the maximum regret. The final alternative decision rule is a probability-based one, the expected value approach. This uses expectations of the likelihood of outcomes to weight the outcomes. The rule is then to choose the option with the highest expected value.

While they are simple and easy to use, decision matrices have some clear drawbacks. Not least amongst these is the necessity of identifying all the possible options and states of nature. With a complex decision there may be many hundreds of possible options and alternative states of nature. It may be possible to aggregate these into a smaller number but there is still the requirement to identify for each the value of the outcome - and these may be highly uncertain. Also the different decision rules may offer different solutions and so it is not clear which is the one to choose. Further, decision matrices are static. That is, they ignore the sequential nature of many decisions, where choices made at one point may alter the probabilities of outcomes happening or the values attached to their consequences. Decision trees offer the ability to take this temporal aspect into account.

4. Decision trees

Decision trees model decisions by identifying the sequence of events in the decision, the decision points or choices, uncertain events and the outcomes. Decision trees represent each decision and state of nature in a branch formation [11]. The first set of branches indicate the options available in the first decision. From these the second set of branches identify the states of nature possible for each of the options. This progression is continued until the final outcomes are reached and no more decisions or alternatives are possible. Values are attached to each outcome and probabilities given for each state of nature. Finally, the decision tree is rolled back from its branches to its root, calculating at each decision point or uncertain event the best branch to follow. Ultimately this should identify the best initial decision. Decision trees have more potential than decision matrices to represent complex decisions but, once again, they require that alternatives and outcomes are identified and that appropriate values and probabilities are attached.

B. Qualitative approaches

Qualitative approaches are essentially non-mathematical. That is not to say that they do not use mathematical tools to assist in the development or the comparison of alternatives but that they do not rely on an explicit mathematical formulation which defines exactly how the decision is modelled. Although there are a number of potential candidate qualitative approaches most can be subsumed into a generalized discussion of scenarios.

1. Scenarios

The most frequently used business approach to modelling an uncertain future is scenario planning. Scenario planning is the process of constructing alternative futures for a businesses external environment. The goal is to learn to use these alternative futures to test the resilience of the current plan of action or to assess how current decisions may affect the long term performance of an organization. Its chief benefits are that it does not rely on a single view of the future and that its output is usually a narrative rather than a single number.

Building scenarios [13] requires, first, an explicit identification of objectives or mission. It is assumed that these are goals which are relatively stable. The next step is to consider the planning horizon - this is typically from a few months to a few years. Clearly, in most cases, the longer the planning horizon the more uncertain the future will become. The third step is retrospective. Looking back to the past may identify what sort of changes have occurred and what drove these changes. This may give insight into trends. The fourth step is to identify what will definitely occur within the planning horizon. This may range from a government election to a product price increase that is already decided upon.

The next step is to attempt to identify the key variables that will have significant consequences. These may be the actions of competitors, the outbreak of war (or peace), or the implications of impending legislation. It may be sensible to locate the probability and the impact of the variables on a two-by-two or three-by-three matrix where the dimensions for each range from low to high. Those variables which have a low impact and a low probability of occurring can be set aside. Attention must be paid to those which have high probabilities and high impacts but those which have high impacts but low probabilities cannot be ignored (a nuclear incident, for example). Scenario planning does not usually attempt to quantify the probabilities and impacts beyond the simple categorization described here. However, in assessing impacts and probabilities it will be necessary to canvass opinion as widely as possible. This step may be made more manageable by aggregating variables where possible. It is also necessary to distinguish between controllable and uncontrollable variables and those that are dependent and those that are independent. To simplify the process, it may be possible to remove or aggregate dependent variables.

The actual scenarios are built by selecting a value for each key variable and assessing the resultant interactions between the key variables, the dependent variables and the assumptions. The development of a narration or story for each future is also helpful. The temptation to develop hundreds of scenarios must be resisted - indeed, the ability afforded by modern computing power to analyze numerous scenarios has increased the tendency for 'paralysis by analysis'. A small number of scenarios, two or three is recommended. It is at this point that the objectives stated in the first step come into play. A strategy for each scenario needs to be developed to try to achieve the objectives under the different possible futures. Clearly, strategies that are flexible enough to achieve the objectives under a number of different scenarios are valuable, and it may be that such a strategy is preferable to one which is optimal but only for a single potential future.

Hence scenarios address uncertainty by identifying a range of possible future situations and looking for actions which take account of these. They are a response to a feeling that large quantitative models are often 'houses of cards' built on dubious assumptions or very uncertain data. Conversely, scenarios are only useful if they can be envisaged and if there is a sensible basis for choosing between those developed.

C. Hybrid approaches

There is a shortage of methods and techniques that allow one to deal with uncertain propositions such as: what is the most probable cause of a certain event? What is the diagnosis given certain pieces of evidence? Or what are the possible explanations for some results obtained? Tversky and Kahneman [14] suggest that 'Most inferences in everyday life rely on models or schemas which are imprecise, incomplete and occasionally incorrect'. The most important approach for dealing with imprecision and incompleteness, both at the modelling level as well as at the solving level is the fuzzy approach [15].

Fuzzy set theory's main objective is to solve problems in which descriptions of activities and observations are imprecise, vague and uncertain [15]. It is a theory of graded concepts where everything is a matter of degree. The term 'fuzzy' refers to situations in which there are no well-defined boundaries, as for example the concept of a 'small number' which can be applied to integer number below, say, 3. Formally, if **X** is a collection of objects denoted generically by **x** then a fuzzy set **A** in **X** is a set of ordered pairs $\mathbf{A = \{x,\mu_A(x) \mid x , X\}}$, where $\mu_{\mathbf{A}}\mathbf{(x)}$ is called the membership function or grade of membership of **x** in **A**. Another important aspect of decision making is the reasoning process that takes place when a decision is made. When in the presence of imprecise concepts this reasoning is usually denoted as approximate reasoning [16, 17, 18, 19].

In the 1970s, Bellman and Zadeh [20] introduced the concept of fuzzy sets to decision making in a fuzzy environment. They define a fuzzy decision as the intersection of fuzzy goals and fuzzy constraints and the optimal decision is the alternative with the maximum membership. In general, assuming there is a collection of alternatives and a set of criteria to classify them, the fuzzy decision is the intersection of the degrees to which alternative A satisfies all criteria. It should be noted that a fuzzy approach can deal both with qualitative and quantitative data, and this is why here it is termed a hybrid approach.

In short, a fuzzy set approach to fuzzy decision making must, essentially, cover the following aspects:

Fuzzification/ Defuzzification. This involves a mechanism responsible for the translation of concepts, such as, for example, 'cheap', into an appropriate fuzzy representation to be handled by the approximate reasoning module and the re-translation of the results into a form understandable to the decision-maker. The mechanism should include the ability to represent the alternatives, attributes, objects and criteria/ constraints. A general scheme for handling fuzzy objects-types (e.g. price) and sub-types (e.g. cheap) is also important.

Approximate reasoning. This should provide various types of approximate inferencing models, since the user may need to employ and compare results with more than one type of reasoning. The versatility of this reasoner is part of the simulation ability the system should contain.

Natural language consultant. It is important to provide query facilities, in natural language format (ie pseudo English), because it allows the decision maker to learn and handle more details about the problem. Illustrating, queries such as 'is it true that most cars are fast?' provide a truth measure regarding the number of existing cases in the database that fulfill the query.

V. Types of Decision Making Problems in Complex Environments

There is a lack of agreed terminology for expressing and describing the components of a decision problem. Some of the more commonly used terms are: objectives, goals, targets, criteria, measures, attributes, features, properties, alternatives and objects. Explicit definitions are given in [21, 22, 23]. Here, the main concepts used related to decision making are: objectives or goals, criteria or features, alternatives and objects. *Alternatives* are the actions, physical objects, strategies etc. that constitute the items of choice. *Attributes/criteria*, are the properties characterizing each alternative, for example, horsepower or price of a vehicle. *Objectives/goals* are collections of attributes available to classify or rate the alternatives. *Objects* represent single concepts (e.g. cheap is an item of attribute price) or instantiations of variables (e.g. $100 (price) or married (status)).

In general, determining the type of decision problem is an important consideration for its modelling and resolution. Many methods and approaches are problem dependent and, therefore, the type of decision situations (problems) is an important issue to be clarified.

A. Multiple criteria decision making.

MacCrimmon [24] presents a comprehensible, systematic distinction between multiple objectives of decision making and multiple attributes which characterize the respective alternatives. He distinguishes three main types of problems (but also stresses that the terminology is often used interchangeably): multiple attribute decision making (MADM); multiple objective decision making (MODM); multiple criteria decision making (MCDM). Typically, multiple attribute and multiple objective decision making can be grouped under the name multiple criteria decision making.

1. *Multiple attribute decision making* (sometimes also called multiple criteria). Multiple attribute decision problems usually refer to a process of choosing from among a set of alternatives in the presence of multiple classifying attributes. The main emphasis is on establishing the preferences, defined by the decision maker, over the criteria in order to evaluate the set of possible alternatives and choose one. An example could be to select a car based on attributes speed, price, safety and comfort or to choose an employee based on his/her experience, skills and maturity. MADM resolution methods consist of two main phases, rating and ranking of the alternatives. This is a qualitative approach due to the existence of criteria subjectivity.

Zimmermann [25] suggests that multiple attribute problems have discrete decision spaces.

Formally, a multiple attribute problem is usually defined as: let $\mathbf{A_{1,..,}A_n}$ be a set of alternatives to be assessed by attributes (criteria) $\mathbf{C_{1,..,}C_m}$; and let $\mathbf{R_{ij}}$ be the numeric rating of alternative $\mathbf{A_i}$ for criteria $\mathbf{C_j}$ and the relative importance of each criterion, $\mathbf{W_j}$. Then the decision function is $\mathbf{D(A_i) = \_ \ R_{i1,...,}R_{ij})}$ where _ represents the aggregation operator (commonly, the intersection min operator) and $\mathbf{D}^*(\mathbf{A_k})$ = max $\mathbf{D}(\mathbf{A_i})$ is the best decision alternative. As can be observed the MADM performs two different tasks, first it rates each alternative and then ranks them to obtain the best one.

Fuzzy multiple attribute decision making, FMADM, includes: the alternatives are the same, $\mathbf{A_{1,..,}A_n}$; there will also exist criteria $\mathbf{C_j}$ but these can be either crisp or fuzzy; the performance rating, $\mathbf{R_{ij}}$, for alternative $\mathbf{A_i}$ with respect to attribute $\mathbf{C_j}$ can be either fuzzy or crisp; the relative importance of attributes can also be either crisp or fuzzy. Many resolution methods have been proposed in the literature to solve FMADM problems [23, 26, 27].

2. *Multiple Objective Decision Making.* Multiple objective decision problems usually consist of optimizing a set of goals subject to a set of constraints. The solution is highly dependent on the constraints, and on satisfying possibly conflicting goals. In this type of decision problem the alternatives can be described both in terms of their attributes (constraints) and in terms of the extent to which they satisfy the objectives This may be described as a quantitative approach because, usually, the resolution involves mathematical optimization methods. An example of this type of problem is to combine the objectives of minimizing the cost and maximize the efficiency of manufacturing cars, considering a set of potential manufacturing processes (alternatives). Zimmermann [25] posits that usually multiple objective problems have continuous decision spaces.

Symbolically, let $\{\mathbf{Z_1}, \mathbf{Z_2}, ..., \mathbf{Z_n}\}$ be a collection of objective functions such as the maximization of profits or the minimization of costs, and a set of constraints representing a relation between the resources needed for each variable, **a**, and the total available, **b**, for the constraint $\mathbf{ax} \leq \mathbf{b}$. The aim is to find the optimal solution that maximizes or minimizes the objective functions subject to satisfying the constraints. It should also be noted that the multiobjective decision problem is a generalization of the single objective one.

The first extension of a crisp optimization problem to fuzzy optimization was proposed by Zimmermann [23, 28], but he considers the problem of fuzzifying the goals and resources limits. In general, fuzzy optimization problems depict their fuzziness in the coefficients, in the parameters, in the goals or in all three. Therefore, the fuzzification of the single or multiple objective linear programming model usually includes four forms of imprecision [29] (more subtle distinctions are made in [30]): 1. Problems with fuzzy constraints; for example, 'the total distance time between warehouses should be considerably fewer than 4 hours'; 2. Problems with fuzzy objectives (goals imposed on the objective functions). For instance, 'the total cost for the project should be kept well below $100,000'; 3. Problems with fuzzy coefficients on the variables, for example, 'the transportation cost per item (x) is about $10'; 4. Combinations of the above. Many solving methods for this type of optimization problem have been proposed in the literature [23, 27, 30, 31].

Summarizing, frequently one does not initially possess a rich understanding of the context-domain and respective attributes of a given problem. Modelling real life situations often requires conceptualizing incomplete or vague terms or concepts. Moreover, the set of alternatives, criteria or constraints and goals do not, necessarily, have a clear-cut nature and sometimes even present some similar or over-lapping characteristics. Therefore, both approaches face the challenge of how to handle fuzzy and/or incomplete information.

C. Group decision making

Most decisions are taken not by individuals but by groups of people. The introduction of more than one person changes the level of uncertainty involved in any decision. On the one hand, groups, as compared to individuals, tend to make higher quality decisions since they have a greater input of skills and abilities. Conversely, however, the need to reach a consensus and the problem of actually eliciting different members' needs, desires and hidden agendas hightens uncertainty. The literature is divided as to whether groups are more or less risk averse than individuals.

Group decision making should be viewed as a social process which transforms a collection of individual decisions into a joint course of action. The essential differences between an individual decision making and a group decision making are in the preference ordering because the objectives and valuations may vary from one decision maker to another, and the information upon which each decides can differ from that of others since the degrees of knowledge are usually not the same.

In general, n-person decision theories are grouped into: group decision making; team theory and n-person game theory. While the last two models are more concerned with maximizing gains (utility function) here the concern is more to study the intrinsic uncertainty in reaching an acceptable group objective and, hence, the focus is on group decision making.

Formally, a group decision consists of a set of n individuals, a set of alternatives $\mathbf{A=\{a_1,...,a_m\}}$; an **n** preference ordering set, **Oi**, in which any alternative from set **A** either individual **k** prefers $\mathbf{a_i}$ to $\mathbf{a_j}$ or the opposite, or s/he is indifferent; and a 'social aggregation function' $\mathbf{F:O_1 x O_2 x....x O_n}$ which associates all individual preference orderings to achieve a group decision 'consensus'.

When classification criteria have a fuzzy nature to determine individual preference, orderings can be complicated. This process is further exacerbated when there are n-individual decision makers stating their preference orderings that must then be aggregated.

A common approach to studying imprecision in group decision making, denoted fuzzy group decision making [32], proposes defining each preference ordering **O:AxA-> {0,1}** as a binary reflexive, anti-symmetric and transitive fuzzy relationship. This fuzzy relation associates with each pair (ai,aj) a grade of membership **μ(ai,aj)** in the social preference ordering, such that μ_R**:AxA -> [0,1]**. The proposal proceeds to define an aggregation function for all individual ordering preferences based on alpha-levels sets with the interpretation that an alpha-set represents an agreement level in the group. That is, the alpha-set **μ** $\geq$ **alpha** represents the threshold of acceptance by all decision makers for the preference orderings of alternatives.

VI. Conclusions

Decision making is complicated by uncertainty. This uncertainty may take many forms and be about many things. There are many approaches to dealing with uncertainty, by reducing it, modelling it or ignoring it. The mere existence of such a plethora of approaches signals that the choice of the appropriate one is contingent. It is contingent on issues such as the nature of the decision, the availability of data, the skills and preferences of the decision maker, the importance of the decision, the time and resources available and so on. A mixed approach, however, using a variety of methods has merit in providing triangulation on the problem by providing alternative views and solutions.

References

[1] Simon, H.A., Models of Man, Wiley, New York, 1957.

[2] Bazerman, M., Judgment in Managerial Decision Making, John Wiley, New York, 1990.

[3] French, S., Decision Theory: An Introduction to the Mathematics of Rationality. Ellis Horwood Limited, 1986.

[4] Combs, C. H., Dawes R. M. and Tversky A., Mathematical Psychology - An Elementary Introduction', Prentice-Hall, 1970.

[5] Starr , M. K. and Zeleny M., Studies in the Management Sciences - Multiple Criteria Decision Making, 1977.

[6] Knight F., Risk, Uncertainty and Profit, Houghton Mifflin, 1921.

[7] Graham, I. and Jones P. L. Expert Systems Knowledge, Uncertainty and Decision. Chapman And Hall Computing, 1988.

[8] Ribeiro, R. A., Powell P. and Baldwin J.F., Uncertainty in Decision Making: An Abductive Perspective" Decision Support Systems, February, 1995.

[9] Tversky, A. and Kahneman D., Judgement Under Uncertainty: Heuristics and Biases, Science, 32-39, Sept., 1974.

[10] Klir G., and Folger T., Fuzzy Sets, Unceratinty and Information, Prentice Hall, 1988.

[11] Cooke S. and Slack N., Making Management Decisions, Prentice Hall, London, 1984.

[12] Townsend E., Investment and Uncertainty, Oliver and Boyd, Edinburgh, 1968.

[13] Linneman R. and Kennell J., Shirt-sleeve Approach to Long Range Plans, Harvard Business Review, 141-150, March 1977.

[14] Tversky, A. and Kahneman D., Rational Choice and the Framing of Decisions. in Readings in Uncertain Reasoning. G. Shafer and J. Pearl (eds), Morgan Kaufman Publisher, Inc. San Mateo, 91-104, 1990.

[15] Zadeh, L. A. Fuzzy Sets Information and Control, 8:338-353, 1990.

[16] Mántaras, R. L., Approximate Reasoning Models. Ellis Horwood Limited, 1990.

[17] Turksen, I. B. Four Methods of Approximate Reasoning with Interval-Valued Fuzzy Sets International Journal of Approximate Reasoning, 3:121-142, 1989.

[18] Zadeh, L. A., Fuzzy Logic and Approximate Reasoning, Synthese, 30:407-428, 1975.

[19] Zadeh, L. A., A Theory of Approximate Reasoning, in Fuzzy Sets and Applications: Selected Papers, L.A. Zadeh. R. Yager, S.Ovchinnikov, R.Tong and H. Nguyen (eds), John Wiley & Sons, 149-194, 1979.

[20] Bellman, R. E. and Zadeh L. A. Decision-Making in a Fuzzy Environment" Management Science 17(4):141-164, 1979.

[21] Starr, M. K. and Zeleny M., MCDM - State And Future Of The Arts TIMS Studies in the Management Sciences, 6:5-29, 1977.

[22] Zeleny, M., Multiple Criteria Decision Making. McGraw-Hill, 1982.

[23] Zimmermann, H.-J., Fuzzy Sets, Decision Making, and Expert Systems. Kluwer Academic Plublishers, 1987.

[24] MacCrimmon, K. R., An Overview of Multiple Objective Decision Making, in Multiple Criteria Decision Making, J. L. Cochrane and M. Zeleny (eds), University of South Carolina Press, 18-44, 1973.

[25] Zimmermann, H.-J., Modelling Flexibility, Vagueness and Uncertainty in Operations Research, Investigación Operativa, 1:7-35, 1988.

[26] Chen, S.-J. and Hwang C.-L., Fuzzy Multiple Attribute Decision Making. Springer-Verlag, 1992.

[27] Kickert, W. J. M., Fuzzy Theories on Decision Making. Martinus Nijhoff Social Sciences Division, 1978.

[28] Zimmermann, H.-J., Fuzzy Programming and Linear Programming with Several Objective Functions, Fuzzy Sets and Systems, 1:45-55, 1978.

[29] Fredizzi, M., Kacprzyk J. and Verdegay J. L., A Survey of Fuzzy Optimization and Mathematical Programming, in Interactive Fuzzy Optimization, M. Fedrizzi and Kacprzyk (eds), Springer-Verlag, 15-28, 1991.

[30] Lai, Y.-J. and Hwang C-L., Fuzzy Multiple Objective Decision Making. Springer-Verlag, 1994.

[31] Pires, F. Pires Moura M.J. and Ribeiro R., Solving Fuzzy Optimisation problems: Flexible Approaches using Simmulated Annealing, Proceedings of ISSCI´96, 1996.

[32] Blin, J. M. and Whinston A.B., Fuzzy Sets and Social Choice Journal of Cybernetics, 3(4):28-36, 1974.