Benefits of Full-Reinforcement Operators for Spacecraft Target Landing

Rita A. Ribeiro, Tiago C. Pais, and Luis F. Simões

Abstract. In this paper we discuss the benefits of using full reinforcement operators for site selection in spacecraft landing on planets. Specifically we discuss a modified Uninorm operator for evaluating sites and a Fimica operator to aggregate pixels for constructing regions that will act as sites to be selected at lower spacecraft altitude. An illustrative case study of spacecraft target landing is presented to clarify the details and usefulness of the proposed operators.

1 Introduction

This paper discusses the suitability of full-reinforcement aggregation operators [1-3] for evaluating alternatives in multicriteria dynamic decision processes. Dynamic multicriteria decision making has been studied, from several different points of view [4-6], but here we focus on discrete spatio-temporal decision processes that involve feedback information for each step.

Moreover, this work extends a preliminary work by the authors [7], to highlight the benefits of using full reinforcement operators to aggregated past and current information in spatio-temporal decision making processes.

The choice of aggregation operator is extremely important in any decision process [8-9], particularly in dynamic decision processes, since they imply changes in input data, over time, as well as feedback from previous steps [10]. In this work, instead of using operators for aggregating criteria we focus on aggregation of alternative ratings at step n with respective feedback from past iteration $n-1$ (i.e. discrete spatio-temporal decision process). Furthermore, we also discuss reinforcement operators to aggregate several alternatives into a single one to create regions, which will be evaluated as alternatives at lower altitudes (i.e. spacecraft's size at low altitude is bigger than a single site, hence regions become alternatives). To this aim we present two variations for well known classes of reinforcement operators, UNINORM and FIMICA [2-3]. We first present the minimal Uninorm operator and a

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Uninorm extension called Hybrid-Reinforcement (HR). Afterwards, we introduce two functions for Sum and Product Fimica aggregation operators.

Most aggregation/rating methods are only either upward reinforcement methods (e.g. Hamacher and Dubois & Prade union operators) or downward methods (e.g. Hamacher and Dubois & Prade intersection operators). When we combine these two concepts we achieve what is called full reinforcement behavior [11]. In this work we highlight why full reinforcement operators are important for dynamic decision processes with feedback.

The case study goal, used to illustrate the suitability of reinforcement operators, is to recommend an adequate interplanetary spacecraft target-landing site [12]. The site adequacy is evaluated with respect to a set of requirements: (1) the site should be safe in terms of maximum local slope, light level and terrain roughness; (2) the site should be reachable with the available fuel; (3) the site should be visible from the camera during the final descent phase.

This chapter is organized as follows. Section 2 describes the case study and also presents the overall dynamic decision process. In Section 3 we briefly describe the UNINORM and FIMICA class of aggregation operators. Afterwards, in section 4, we present a detailed discussion and numerical examples regarding the proposed Uninorm and Fimica based aggregation operators. An assessment concerning the aforementioned operators within the case study is presented in Section 5. Section 6 contains the concluding remarks.

2 Spacecraft Landing Overview

The main objective of a descent phase, in spacecraft landing on planets, is to select the safest site for landing [12-14]. The goal of the case study was to provide an adequate target-landing site, evaluated with a set of requirements, as mentioned in the introduction. The case study was focused in the final descent phase (around 2 Km from surface), when hazard maps can be obtained [12, 14]. To achieve the objectives we had access to simulated hazard maps (images taken by onboard camara) of dimensions 512x512 pixels that provide assessments of terrain features and trajectory constraints on a landing scenario. Notice that some values used in the case study description are just indicative due to reasons of confidentiality.

The seven input criteria, which correspond to the hazard maps obtained during the descent phase, are [14]: slope, texture, fuel, reachability, distance, shadow and scientific interest. The alternatives are the pixels of the combined search space (derived from merging 7 matrices of 512x512, corresponding to the input images/hazard maps), resulting from a data preparation process requiring normalization and data fusion (out of scope here, some details can be seen in [12]) . The resulting set of alternatives is about 260.000 possible alternative sites per iteration (512*512= 262,144).

The dynamic decision process includes around 40-60 iterations, and, for each one, there is an evaluation process (called ranking process), which includes combining the k best alternatives from iteration $n-1$ (historic set feedback) with the current rated ones at the n iteration (for details about this process see [14]). We only consider the $n-1$ iteration, as historical information to aggregate with current rating, because we update the historic set per iteration and then pass this information to the next iteration (feedback of the dynamic process). When there are no more iterations the decision process stops and the best alternative is the one with the highest combined rate (after combining historic and current rating). For the dynamic evaluation process we used a Uninorm based operator, which we called Hybrid Reinforcement (HR) Operator.

Another important aspect to take in consideration in the dynamic evaluation process is the relative proximity of the spacecraft to planet surface. When the spacecraft altitude is high the current rating refers to pixels in the images (corresponding to site coordinates). However, in the final stages of landing on a planet, we have to consider that the spacecraft size is larger than the image pixels; hence instead of selecting single pixels (coordinates) we have to select regions (i.e. sets of single pixels) for landing. Details about the regions aggregation process are given in [13]. For the regions evaluation process we defined a Product FIMICA operator, which proved to be appropriate for aggregating the pixels into regions.

Figure 1 depicts the dynamic decision process of the case study.

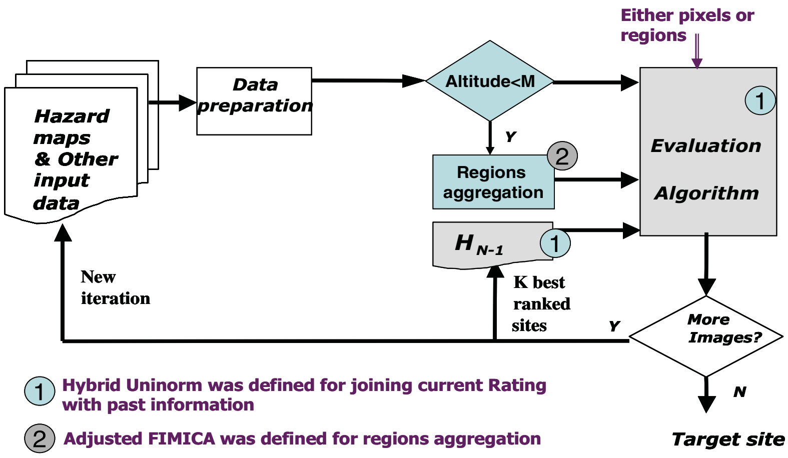


Fig. 1 Dynamic decision process of Spacecreaft Landing case study

In summary, the illustrative case study includes historical information from previous iterations (feedback) and uses suitable adaptations of full reinforcement operators in a dynamic decision process. Moreover, when spacecraft altitude is low a FIMICA reinforcement operator was devised to aggregate pixels into regions, the latter becoming alternatives of the multi-criteria dynamic decision system.

3 Background on Full Reinforcement Operators

Aggregation operators have been extensively studied in the literature and their usage in fuzzy multi-criteria problems is widely spread (see for example [8, 15-18]).

In this work we focus on aggregation operators with full-reinforcement behaviour, because this is an important quality for dynamic decision processes. Specifically, we discuss the Uninorm and Fimica classes [1-3] as beneficial for our type of problem. Full-reinforcement property means that: a) for a set of high scores to "positively" reinforce each other, it must obtain a higher score than any of the elements alone (upward reinforcement); b) for a set of low scores to "negatively" reinforce each other it must obtain a lower score than any of the elements alone (downward reinforcement). The main difference between the Fimica and Uninorm operators is that the former can be continuous.

3.1 UNINORM

The *UNINORM* class of aggregation operators was introduced by [1-2] as a generalization of T-norms and T-conorms. One of the main characteristics of this operator is the consideration of a neutral element, anywhere in the interval]0, 1[. A uninorm R is a mapping

$$R : [0,1]^2 \rightarrow [0,1] \tag{1}$$

having the following properties:

$$R(a,b) = R(b,a) \quad \text{(commutativity);} \tag{2}$$

$$R(a,b) \geq R(c,d) \quad \text{if} \quad a \geq c \text{ and } b \geq d \quad \text{(monotonocity);} \tag{3}$$

$$R(a, R(b,c)) = R(R(a,b),c) \quad \text{(associativity);} \tag{4}$$

There exist some elements $e \in]0,1[$ called the neutral element such that for all $a \in [0,1]$, $R(a,e) = a$.

Moreover, uninorm operators also present a compensatory behaviour, i.e., any Uninorm R satisfies:

$$\begin{array}{l} \min(x,y) \leq U(x,y) \leq \max(x,y) \ , \ (x,y) \in [0,e[\times]e,1] \cup]e,1] \times [0,e[\\ \text{where } e \text{ is a neutral element, i.e., } \exists e \in]0,1[\ \ \forall x \in [0,1] : U(x,e) = x \end{array} \tag{5}$$

It is known that any Uninorm operator have at least a discontinuity in $(1,0)$ and $(0,1)$. Therefore, uninorm operators may present asymptotic behaviour, i.e., a small change in the arguments implies a significant change in output value (cfr. Section 4.5). For example, the minimal uninorm operator [11] has discontinuities in $\{e\} \times [e,1] \wedge [e,1] \times \{e\}$, as shown in Figure 2.

3.2 FIMICA

The *FIMICA* class of aggregation operators [3] were derived from the MICA operators [15]. They are defined as a bag mapping:

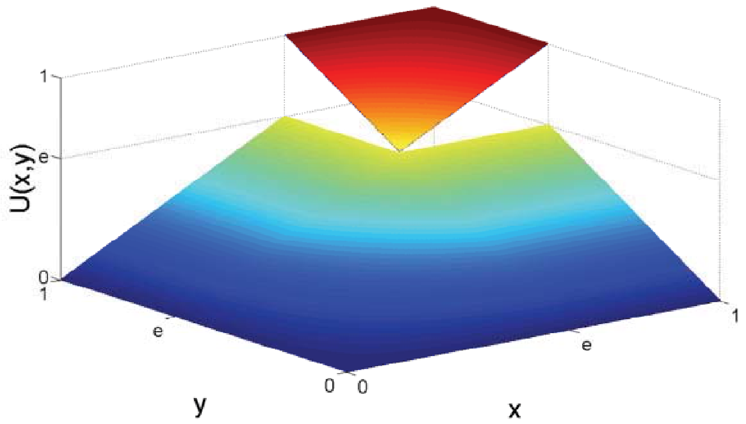


Fig. 2 Plot of minimal Uninorm (where $e = 0.65$ is the neutral element)

$$F : U^I \rightarrow I \tag{6}$$

having the following properties:

$$\text{If } A \geq B \text{ then } F(A) \geq F(B) \qquad \text{(monotonocity);} \tag{7}$$

$$\exists g \in I \quad \forall A \in U^I : \; F(A \oplus \langle g \rangle) = F(A) \qquad \text{(fixed identity);} \tag{8}$$

where $g \in]0,1[$ is called identity element, A and B are any bag, $A =< a_1,...,a_n >$, $a_i \in I = [0,1]$ and $B =< b_1,...,b_n >$, $b_i \in I = [0,1]$.

Fimica operators present full reinforcement behaviour, similar to Uninorm operator, and the F operator is also monotonic and commutative with respect to arguments in A. In addition, an important aspect of Fimica is the choice of an appropriate function, F, to control the operator behaviour, e.g., deciding if a small change in the arguments does (or not) imply a significant change in output value (cfr. Section 4.5).

4 Proposed Adjustments for Reinforcement Operators

In this section we discuss four variations for full reinforcement operators. First, we present the minimal Uninorm. Second we introduce one operator, called Hybrid Reinforcement (HR) operator, which is based on the minimal Uninorm [11] and includes a compensatory nature in all four quadrants of the image space (see Figure 2 and Figure 3). The third and fourth variations belong to the additive and product family of Fimica class aggregation operators, respectively, where particular functions were devised to better fit the illustrative example. All variations are discussed within the context of a small example and the illustrative case study. The authors presented a preliminary version of some of these operators in [7].

4.1 Minimal Uninorm

The proposed minimal Uninorm operator is formally defined as follows.

$$U_{\min}(x, y) = \begin{cases} \eta.T(\frac{x}{\eta}, \frac{y}{\eta}) & , \quad \text{for } (x \le \eta \wedge y < \eta) \vee (x < \eta \ \wedge \ y \le \eta) \\ \eta + (1-\eta).S(\frac{x-\eta}{1-\eta}, \frac{y-\eta}{1-\eta}), & \quad \text{for } x \ge \eta \text{ and } y \ge \eta \\ \text{Min(x, y)} & , \quad \text{elsewhere} \end{cases} \tag{9}$$

where:
η is a neutral element;
T-norm is Hamacher intersection operator (T);
S- norm is Hamacher union operator (S);

The neutral element η is the parameter influencing the quantity of upward or downward reinforcement operations. In our case we use quantiles for neutral element because with a high quantile we ensure the majority of values fall before the bounded quantile value, hence more downward reinforcement operations. Using a lower quantile we ensure more upward reinforcement operations in the aggregation of alternative rating at iteration n with historic value from iteration *n-1*.

For S-norm and T-norm (S, T) we use the following Hamacher operator formulas [9]:

$$S_\alpha(x, y) = \frac{x + y - (2-\alpha) * x * y}{1 - (1-\alpha) * x * y} \quad , \quad \text{where } \alpha \in [0;+\infty[\text{ and x, y} \in [0;1] \tag{10}$$

$$T_\alpha(x, y) = \frac{x * y}{\alpha + (1-\alpha)(x + y - y * x)} \quad , \quad \text{where } \alpha \in [0;+\infty[\text{ and x, y} \in [0;1] \tag{11}$$

In our case we use a low value for parameter α because we want to reward or penalize the rating values smoothly instead of using an aggressive aggregation behavior. The choice of Hamacher operators for the upper and lower reinforcement was based on its synergetic nature.

4.2 HR Operator

The motivation for this adapted operator was a need for an aggregation operator with full reinforcement characteristics, but flexible enough to include an averaging compensatory nature in the interval, $]0,\eta[\times]\eta,1[\cup]\eta,1[\times]0,\eta[$. Hence, we proposed an adaptation of the minimal Uninorm operator [14] which includes

Hamacher synergetic operators [9] for intersection and union and OWA [16], in the interval $]0,\eta[\times]\eta,1[\cup]\eta,1[\times]0,\eta[$. This latter interval is the one outside the image space of t-norms and s-norms, since these are bounded by the neutral element η (see Figure 3 a.). Formally, the proposed hybrid reinforcement operator, HR, is a mapping $[0,1]\times[0,1]\rightarrow[0,1]$, such that,

$$HR(x,y)=\begin{cases} \eta.T(\frac{x}{\eta},\frac{y}{\eta}) & \text{, for } (x\le\eta \wedge y<\eta)\vee(x<\eta \ \wedge \ y\le\eta) \\ \eta+(1-\eta).S(\frac{x-\eta}{1-\eta},\frac{y-\eta}{1-\eta}) & \text{, for } x\ge\eta \text{ and } y\ge\eta \\ \mathrm{OWA(x,y)} & \text{, elsewhere} \end{cases} \tag{12}$$

where:
η is a neutral element;
T-norm represents Hamacher intersection operator (T) (cfr. Equation 10);
S- norm represents Hamacher union operator (S) (cfr. Equation 11);
OWA represents Yager´s OWA operator (cfr. Equation 13).

For OWA aggregation operator [16] we use the following formulation:

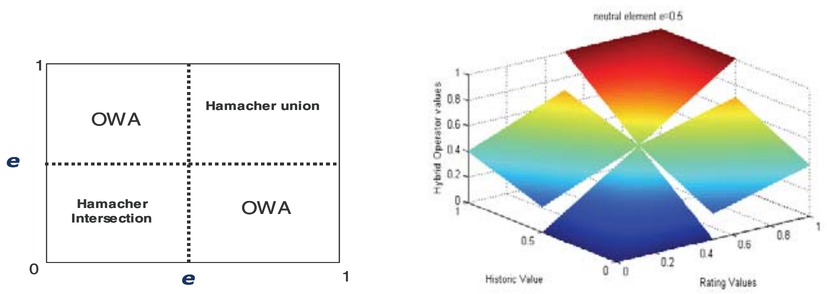
$$\begin{aligned} &OWA(x,y)=w_1\times\max(x,y)+w_2\times\min(x,y), \\ &\text{where } w_1+w_2=1 \quad \text{and} \quad \mathrm{x,y}\in[0;1] \end{aligned} \tag{13}$$

The weights for the OWA aggregation operator (w_1 and w_2) will have in consideration that giving more weight to lower values will decrease the aggregation value; this is what we are looking to avoid selecting sites with lower rating values.

A similar argument as the one presented in section 4.1 can be made regarding the choice of Hamacher intersection and union functions for S and T-norm, respectively. Moreover, the same parameters were also used as in the minimal Uninorm operator.

The proposed *HR* operator is suitable for our case because our goal is to combine historical information, from previous iteration ($H_{n-1}\Rightarrow x$), with current rating value ($R_n \Rightarrow y$), until we reach a conclusion (stopping criterion). Combining current and feedback information is the feedback process in the dynamic decision process.

In summary there were four main requirements for the *HR* operator: to ensure full reinforcement capability; to include full compensatory nature in all quadrants; to take in consideration the order of elements; to ensure synergy between arguments by using Hamacher operators for T-norm and S-norm. Figure 3 (a) summarizes the combination of operators for each image space quadrant, determined by the neutral element; and plot (b) represents its behaviour for a neutral element of 0.5.



a) HR image space b) Plot example for e=0.5

Fig. 3 Search space of HR operator and a behavioural example.

The algorithm parameters were tuned to get a coherent behaviour for our case study. The neutral element η was set to a high quantile of R_n to obtain a small subset with high classifications. Then, we determined the α parameter for both Hamacher operators (S and T equations), and weights used for the OWA aggregation operator.

The last step of pixels evaluation (phase where spacecraft is at relative high altitude) is to rank the values decreasingly and, from this ordered list, we select a sub-set for the next iteration (historic set). At each iteration n we select the k best ranked sites and, depending on the altitude from the planet surface, the historical set size k varies. With this procedure, it can happen that the best choice of alternative is not the highest regarding its rating value, in the respective iteration. This situation is due to the use of historical feedback information and the behaviour of the hybrid reinforcement operator (HR, eq. 12) in the computation of the dynamic decision model. We want to select sites that proved to be good during a certain period of time, i.e. that provide some consensus about its suitability!

Finally, it is important to highlight that HR operator does not fulfil all properties of Uninorm operators, specifically the associativity condition. However, since for any iteration we just aggregate two values, historical and current rating values, there are no associative problems in our case. All other properties of Uninorm operators are satisfied on $[0,1]\times[0,1]$.

4.3 Additive FIMICA Operator

The additive family of the FIMICA class of aggregation operators is defined as follows [3].

$$
\begin{aligned}
& S(A) = f\Big(\sum_{i\in I}(a_i - g)\Big) \\
& \text{where: } f : D \subseteq \Re \to [0,1]; \\
& \qquad A =< a_1,\cdots,a_n > \text{ is a bag defined over the unit interval;} \\
& \qquad g \in [0,1] \text{ is the identity.}
\end{aligned}
\tag{14}
$$

Several f functions were tested, for our illustrative case study, and we chose the most suitable to ensure a smooth behaviour. Formally, the chosen function is,

$$S(<x,y>) = 0.5 - \frac{\arctan((x-g)+(y-g))}{\pi} \quad \text{, where } x, y \in [0,1] \tag{15}$$

In Figure 4 we show its respective plot.

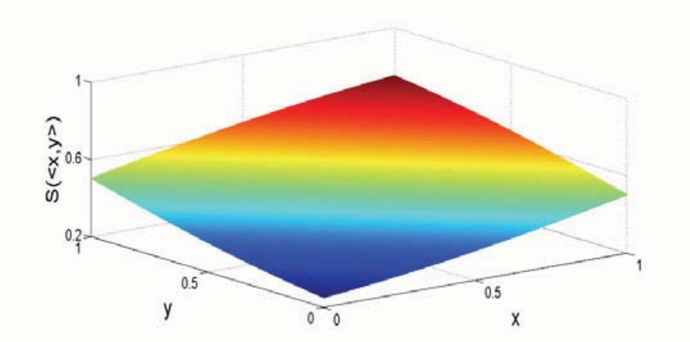


Fig. 4 Plot of additive Fimica operator for g= 0.5.

4.4 Product FIMICA Operator

The product Fimica aggregation operators are defined in a similar fashion as the previous additive Fimica operators. The product operator is defined as follows [3].

$$\begin{aligned} & V(A) = f\left(\prod_{i \in I}\left(\frac{a_i}{g}\right)\right) \\ & \text{where}: f: D \subseteq \Re_0^+ \to [0,1]; \\ & \quad A = < a_1, \cdots, a_n > \text{ is a bag defined over the unit interval}; \\ & \quad g \in]0,1] \text{ is the identity.} \end{aligned} \tag{16}$$

As in the previous section, several f functions were tested and the most suitable to ensure the necessary behaviour is defined as in Equation (17).

$$V(<x,y>) = 1 - \frac{1}{1 + \frac{x}{g} \times \frac{y}{g}} \quad \text{, where } x, y \in [0,1] \tag{17}$$

In Figure 5 we show its respective plot.

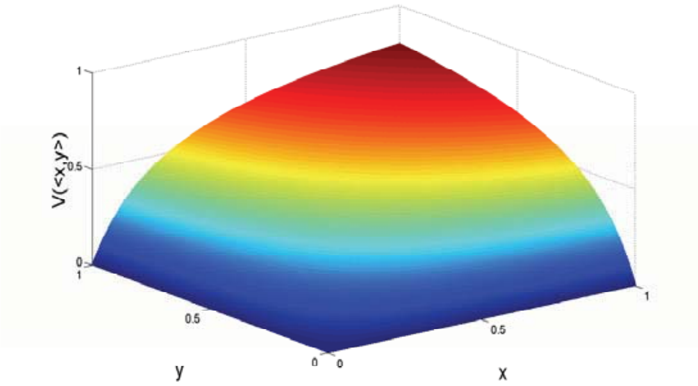


Fig. 5 Plot of product Fimica operator for g=0.5.

Comparing Figure 2 with Figure 5 it is obvious the latter continuous nature and the non-existence of intervals where smooth behaviour is not guaranteed.

4.5 Numerical Examples

This section illustrates the results of applying the proposed adjusted operators, to aggregate 2 criteria (columns) for 3 alternatives (rows), as depicted in the following matrix,

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cc} H_{n-1} & R_n \end{array} \\ \begin{bmatrix} 0.104 & 0.21 \\ 0.2 & 0.11 \\ 0.1039 & 0.21 \end{bmatrix} \end{array}$$

4.5.1 Example 1 - Umin Operator

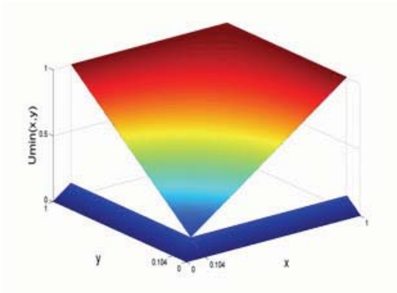
Consider $e = 0.104$ (neutral element), Hamacher's parameters $\alpha = 0.8$. Applying U_{min}, we have:

$$U_{\min A_1}(x_{11}, x_{12}) = 0.2102$$

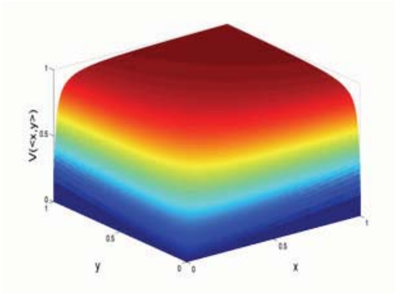
$$U_{\min A_2}(x_{21}, x_{22}) = 0.104 + (1 - 0.104) H_{\cup}\left(\frac{0.2 - 0.104}{1 - 0.104}, \frac{0.11 - 0.104}{1 - 0.104}\right) = 0.2052$$

$$U_{\min A_3}(x_{31}, x_{32}) = \min(0.1039\,, 0.21) = 0.1039$$

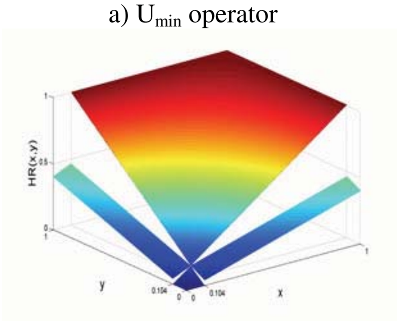
The results are: A1>A2>A3. Figure 6 a) depicts the plot for this operator behaviour.



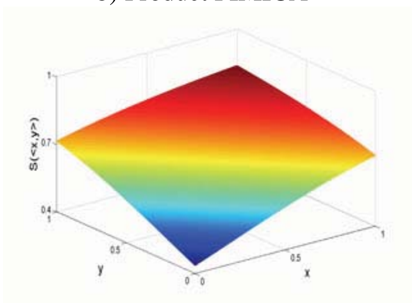
a) U_{min} operator



b) Product FIMICA



c) HR operator



d) Additive FIMICA

Fig. 6 Plots of operators used in numerical examples with e=g=0.104 and α =0.8.

4.5.2 Example 2 - HR Operator

Consider $e = 0.104$ (neutral element), Hamacher's parameters $\alpha = 0.8$ and OWA w_1 and w_2 equal to 0.4 and 0.6, respectively. Applying HR, we have:

$$HR_{A_1}(x_{11}, x_{12}) = 0.2102$$

$$HR_{A_2}(x_{21}, x_{22}) = 0.104 + (1 - 0.104) H_{\cup}\left(\frac{0.2 - 0.104}{1 - 0.104}, \frac{0.11 - 0.104}{1 - 0.104}\right) = 0.2052$$

$$HR_{A_3}(x_{31}, x_{32}) = OWA(0.1039, 0.21) = 0.1463$$

The results are: A1>A2>A3. Figure 6 c) shows the plot for this operator.

4.5.3 Example 3 - Product Fimica

Next we show the results obtained with the proposed product Fimica considering *g = 0.104* for identity element:

$$V_{A_1}(< x_{11}, x_{12} >) = 1 - \frac{1}{1 + \left(\frac{0.104}{0.104} * \frac{0.21}{0.104}\right)} = 0.50476$$

$$V_{A_2}(< x_{21}, x_{22} >) = 0.50836$$

$$V_{A_3}(< x_{31}, x_{32} >) = 0.50429$$

The results are: A2>A1>A3. Figure 6 b) depicts the behaviour for this operator.

4.5.4 Example 4 - Additive Fimica

In this example is illustrated the results obtained with the proposed additive Fimica considering $g = 0.104$ for identity element:

$$V_{A_1}(<x_{11}, x_{12}>) = 0.5 - \frac{\arctan((0.21 - 0.104) + (0.104 - 0.104))}{\pi} = 0.53362$$
$$V_{A_2}(<x_{21}, x_{22}>) = 0.53236$$
$$V_{A_3}(<x_{31}, x_{32}>) = 0.53358$$

The results are: A1>A3>A2. Figure 6 d) depicts the plot of this operator´ behaviour.

4.5.5 Discussion of Examples Results

All reinforcement operators consider that alternative A3 is worse then A1 and this is a coherent result since A3 is dominated by A1.

Comparing Example 1 and Example 2 we can observe that the results are identical in terms of order of importance of alternatives but the Umin does not have a compensatory behaviour when one criteria is good (above the neutral element) and another is bad (below the threshold) hence it reduces the overall importance to its weakest criteria. In our case study we do not want this behaviour because we are aggregating historic information with current information and both should count to the overall result. Hence, for our dynamic model we selected the HR operator.

All operators consider that A1 is the best alternative except product FIMICA that considers A2 as the best alternative. This shows that for aggregating only two criteria this operator might not be the best choice but for aggregating several criteria it is another story. Comparing the results of Example 1 and 3, we can observe that product Fimica operator has a more coherent behaviour in the sense that similar alternatives have very close ratings (A1 and A3). In both operators, if we have one $a_i = 0$, the aggregated value will also be zero. This is a critical feature for our operator since their purpose is to aggregate a set of pixels into a single region. This means that if one region contains an unacceptable pixel the entire region is considered undesirable for landing.

In summary, it seems that HR is a good choice for aggregating two criteria that require synergy and compensatory behaviour (dynamic model), while product FIMICA seems good for aggregating several criteria when we want to ensure a smoother behaviour in the aggregation and the elimination of an alternative if there is one or more “bad” elements in that alternative. The choices of operators for the case study are further discussed in the next section.

In Figure 6 we show the four operators plots, the same neutral element and identity element value was used in all the examples. The differences are obvious in terms of behaviors’ in the different quadrants of the search space.

5 Assessment of Reinforcement Operators within Case Study

In this work we described a dynamic evaluation process for rating sites, either for pixels or regions depending on the spacecraft altitude.

The dynamic decision process is done with the HR operator (12) and applies to either pixels or regions. To construct the regions (grouping of ratings of nearby pixels) we used the product Fimica (17). We used product FIMICA to ensure a smother aggregation behaviour and fulfilment of the associative property, when there are more than 2 elements to aggregate.

The main difference between the U_{min} and the product FIMICA operator is that the former is continuous. Further, choosing an appropriate function (17) guarantees a more smooth behaviour, i.e., a small change in the arguments does not imply a significant change in output values. This was illustrated in the small numerical examples above (Section 3.3), where a difference of only 0.01 (but it could be as small as we wanted) implies a change in the ranking order. When evaluating regions we observed that some had a poor ranking position, even though most of its pixels have high values and only a couple of them had smaller values than the neutral element (but very small differences).

Table 1 depicts results of grouping pixels into regions, for one iteration of the decision process. It can be observed that for the ten best regions, obtained with product FIMICA, the min Uninorm only ranks alternative 4 as the "best" and number 10 as the third ranked. It misses all other good regions that were obtained with product FIMICA. The results were also validated by Space experts and they concurred that FIMICA operator was more suitable to rate regions.

Table 1 Best landing site regions identified in one iteration, using the product FIMICA operator, and results for the same regions using the minimal Uninorm operator

Region 2D coordinates		FIMICA		Uninorm	
x	y	Rating	Ranking	Rating	Ranking
207	267	0,793156	1	0,983649	41
206	267	0,792631	2	0,983618	42
208	267	0,791215	3	0,983521	49
215	250	0,78965	4	0,984586	1
209	267	0,789257	5	0,983384	55
216	250	0,788848	6	0,984528	4
209	266	0,787445	7	0,983346	58
205	267	0,787431	8	0,983331	59
183	257	0,786907	9	0,984029	20
214	248	0,786865	10	0,984543	3

Now we will discuss the operators involved in the dynamic decision process of aggregating past and current information. The HR and additive Fimica operators have a completely different behaviour when compared with the two previous discussed operators. Also, as mentioned, in our case study they have a different purpose. These operators are must more adequate to be used in the dynamic aggregation phase due to their compensatory nature on the entire domain. When we observe the results presented in Section 4.5, again, the (additive) Fimica operator seems to have a more coherent behaviour. However, in this case, the function used in the Fimica operator is much more computational demanding and time consuming than the HR function even though it presents a more smooth behaviour. When we are dealing with dynamical decision models, where several decisions are made until a "consensus" is reached, a major constraint is the computational cost in terms of time. Hence, the HR operator is more suitable for dynamic aggregation. Moreover, for our case study computational time was an essential feature for the feasibility of the entire algorithm, so HR was chosen.

6 Concluding Remarks

In this work we discussed details about full reinforcement operators. Specifically, we focused on a hybrid operator (HR) used in a dynamic decision process for selecting alternatives and a product FIMICA operator used for aggregating (grouping) ratings of alternatives.

The suitability and flexibility of using full-reinforcement operators was assessed with an illustrative example and also with a case study of site selection for spacecraft landing on planets. Specifically the hybrid reinforcement operator was used to combine past and current ratings at each iteration of the dynamic decision process, while the proposed product Fimica was used for aggregating pixels into regions (when a spacecraft is close to the surface it is bigger than pixels in hazard maps, hence we need to select regions). Moreover, since the case study involves a dynamic decision process the usage of full-reinforcement operators proved quite successful for achieving a good decision after several iterations.

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